CORE
Metadata, citation and similar papers at core.ac.uk
Provided by CERN Document Server

hep-th/0612267
TIT/HEP-564
YITP-06-65
December, 2006

arXiv:hep-th/0612267v1 26 Dec 2006

Deformation of $\mathcal{N} = 4$ Super Yang-Mills Theory in Graviphoton Background

Katsushi Ito[1], Yoshishige Kobayashi[1] and Shin Sasaki[2]

[1] *Department of Physics*
Tokyo Institute of Technology
Tokyo, 152-8551, Japan

[2] *Yukawa Institute for Theoretical Physics*
Kyoto University
Kyoto 606–8502, Japan

Abstract

We study deformation of $\mathcal{N} = 4$ super Yang-Mills theory from type IIB superstrings with D3-branes in the constant R-R background. We compute disk amplitudes with one graviphoton vertex operator and investigate the zero-slope limit of the amplitudes. We obtain the effective action deformed by the graviphoton background, which contains the one defined in non(anti)commutative $\mathcal{N} = 1$ superspace as special case. The bosonic part of the Lagrangian gives the Chern-Simons term coupled with the R-R potential. We study the vacuum configuration of the deformed Lagrangian and find the fuzzy sphere configuration for scalar fields.

1 Introduction

Ramond-Ramond (R-R) background in type II superstring theories plays an important role in studying effective theories on the D-branes. In particular constant graviphoton background induces non(anti)commutativity in world-volume superspace [1, 2, 3]. $\mathcal{N}=1$ super Yang-Mills theory on non(anti)commutative superspace ($\mathcal{N}=1/2$ superspace) [4] is obtained from type IIB superstrings with constant graviphoton background compactified on a Calabi-Yau threefold. The deformed action was constructed explicitly in [5] from open string amplitudes in type IIB superstring theory compactified on an orbifold $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_2$ in graviphoton background.

Non(anti)commutative $\mathcal{N}=2$ harmonic superspace provides various types of deformed $\mathcal{N}=2$ supersymmetric gauge theories [6, 7, 8, 9, 10]. In a previous paper [11], two of the present authors studied the deformation of $\mathcal{N}=2$ supersymmetric Yang-Mills theory from the open string disk amplitudes with one graviphoton vertex operator in type IIB superstring theory. The deformed $\mathcal{N}=2$ super Yang-Mills theory is realized as the low-energy effective theory on the D3-branes in the type IIB superstrings compactified on $\mathbf{C}^2/\mathbf{Z}_2$ with constant graviphoton background. The constant R-R backgrounds $\mathcal{F}^{\alpha\beta ij}$ are classified into four types of deformations (S,S), (S,A), (A,S) and (A,A)-types, in which (S,S) and (A,A)-types deformations are related to the deformation of $\mathcal{N}=2$ superspace. By choosing the (S,S)-type graviphoton background and the appropriate scaling condition, it was shown that the effective Lagrangian on the D3-branes becomes the deformed one in non(anti)commutative $\mathcal{N}=2$ harmonic superspace at the lowest order in deformation parameters.

It is an interesting problem to extend this deformation to $\mathcal{N}=4$ case. Since superspace formalism keeping $\mathcal{N}=4$ supersymmetry manifestly is not yet known, superstring approach provides a systematic method for understanding general non(anti)commutative deformation of $\mathcal{N}=4$ theory. The couplings between R-R fields and the world-volume massless fields on the Dp-branes have been studied in [12, 13], which are written in the form of the Chern-Simons action. In recent papers [14], by restricting the constant five-form background to the deformation parameter of $\mathcal{N}=1/2$ superspace, it is shown that the bosonic part of the Chern-Simons action reduces to the deformed interaction terms of the $\mathcal{N}=4$ super Yang-Mills theory in $\mathcal{N}=1/2$ superspace. The interaction terms including fermions are constructed by the supersymmetric completion [14] using remaining supersymmetries but disagree with the deformed action based on non(anti)commutative superspace.

In this paper we will study the deformation of $\mathcal{N}=4$ supersymmetric Yang-Mills theory from open string amplitudes in the constant R-R background. We put the D3-branes in type IIB superstrings in flat ten-dimensional (Euclidean) space-time. We will compute open superstring disk amplitudes with one graviphoton vertex operator and determine the low-energy effective Lagrangian at the first order in the deformation parameter C. We will find the effective action in a special graviphoton background is consistent with the deformed action defined on non(anti)commutative $\mathcal{N}=1$ superspace at the first order in C.

The action contains new scalar potential terms deformed by C. This term corresponds to the Myers term [12] which gives rise to a dielectric configuration for scalar fields. In the $\mathcal{N}=1/2$ superspace formalism, this type of configuration was found in [14]. In the present paper, we will explore the fuzzy two-sphere configuration for the scalar fields in the presence of constant $U(1)$ gauge fields, based on the (S,S)-type deformed action.

This paper is organized as follows: in section 2, we review D3-brane realization of $\mathcal{N}=4$ super Yang-Mill theory in type IIB superstring theory and classify the R-R background. In section 3, we calculate the disk amplitudes with one closed string graviphoton vertex operator and the effective action on the D3-branes. In section 4, we study the vacuum configuration of the deformed $\mathcal{N}=4$ theory and find the fuzzy two-sphere configuration for scalar fields.

2 D3-brane realization of $\mathcal{N}=4$ super Yang-Mills theory

In this section we review the D3-brane realization of $\mathcal{N}=4$ super Yang-Mills theory [15].

2.1 Type IIB superstrings

We begin with explaining type IIB superstrings in flat space-time. We will use the NSR formalism. Let $X^m(z,\bar{z})$, $\psi^m(z)$ and $\tilde{\psi}^m(\bar{z})$ $(m = 1, \cdots, 10)$ be free bosons and fermions with world-sheet coordinates $(z, \bar{z})$. Their operator product expansions (OPEs) are given by $X^m(z)X^n(w) \sim -\delta^{mn} \ln(z-w)$ and $\psi^m(z)\psi^n(w) \sim \delta^{mn}/(z-w)$. Here the space-time signature is Euclidean. Fermionic ghost system (b, c) with conformal weight $(2, -1)$ and bosonic ghost system (β, γ) with weight $(3/2, -1/2)$ are also introduced. The world-sheet fermions $\psi^m(z)$ are bosonized in terms of free bosons $\phi^a(z)$ $(a = 1, \cdots, 5)$ by

$$f^{\pm e_a}(z) \equiv \frac{1}{\sqrt{2}}(\psi^{2a-1} \mp i\psi^{2a}) =: e^{\pm\phi^a}(z) : c_{e^a}. \tag{1}$$

Here $\phi^a(z)$ satisfy the OPE $\phi^a(z)\phi^b(w) \sim \delta^{ab}\ln(z-w)$ and the vectors e_a are orthonormal basis in the $SO(10)$ weight lattice space and c_{e^a} is a cocycle factor [16]. The bosonic ghost is also bosonized [17]: $\beta = \partial\xi e^{-\phi}$, $\gamma = e^{\phi}\eta$ with OPE $\phi(z)\phi(w) \sim -\ln(z-w)$. The R-sector is constructed from spin fields $S^{\lambda}(z) = e^{\lambda\phi}(z)c_{\lambda}$, where $\phi = \phi^a e_a$ and $\lambda = \frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4 \pm e_5)$. λ belongs to the spinor representation of $SO(10)$. c_λ is a cocycle factor. In type IIB theory, after the GSO projection, we have spinor fields which have odd number of minus signs in λ, for both left and right movers.

We now introduce parallel N D3-branes in the $(x^1, \cdots, x^3)$-directions. Since the D3-branes breaks the ten-dimensional Lorentz symmetry $SO(10)$ to $SO(4) \times SO(6)$, the spin field $S^{\lambda}(z)$ is decomposed as $(S_\alpha S_A, S^{\dot{\alpha}} S^A)$, where S_α and $S^{\dot{\alpha}}$ $(\alpha, \dot{\alpha} = 1, 2)$ are four-dimensional Weyl spinor and S_A and S^A $(A = 1, 2, 3, 4)$ are six-dimensional Weyl spinor. S^A (S_A) belongs to the (anti-)fundamental representation of $SU(4)$. The Dirac matrices for four dimensional part are $\sigma_\mu = (i\tau^1, i\tau^2, i\tau^3, 1)$ and $\bar{\sigma}_\mu = (-i\tau^1, -i\tau^2, -i\tau^3, 1)$, where τ^i $(i = 1, 2, 3)$ are the Pauli matrices. The Lorentz generators are defined by $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$ and $\bar{\sigma}_{\mu\nu} = \frac{1}{4}(\bar{\sigma}_\mu\sigma_\mu - \bar{\sigma}_\nu\sigma_\mu)$.

The gamma matrices for six-dimensional part are given by

$$\Sigma^a = \left(\eta^3, -i\bar{\eta}^3, \eta^2, -i\bar{\eta}^2, \eta^1, i\bar{\eta}^1\right), \quad \bar{\Sigma}^a = (-\eta^3, -i\bar{\eta}^3, -\eta^2, -i\bar{\eta}^2, -\eta^1, i\bar{\eta}^1), \tag{2}$$

where $a = 1, \cdots, 6$. $\eta^a_{\mu\nu}$ and $\bar{\eta}^a_{\mu\nu}$ are 't Hooft symbols, which are defined by $\sigma_{\mu\nu} = \frac{i}{2}\eta^a_{\mu\nu}\tau^a$ and $\bar{\sigma}_{\mu\nu} = \frac{i}{2}\bar{\eta}^a_{\mu\nu}\tau^a$. The matrices (2) satisfy the algebra

$$(\Sigma^a)^{AB}(\bar{\Sigma}^a)_{BC} + (\Sigma^b)^{AB}(\bar{\Sigma}^a)_{BC} = 2\delta^{ab}\delta^A_C. \tag{3}$$

The massless spectrum of open strings contain gauge fields A_μ, six scalars φ^a in the NS sector and gauginos $\Lambda^{\alpha A}$ and $\bar{\Lambda}_{\dot{\alpha}A}$ in the R sector. The vertex operators for gauge and scalar fields in the (-1) picture are given by

$$
\begin{aligned}
V_A^{(-1)}(y;p) &= (2\pi\alpha')^{\frac{1}{2}}\frac{A_\mu(p)}{\sqrt{2}}\psi^\mu(y)e^{-\phi(y)}e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}, \\
V_\varphi^{(-1)}(y;p) &= (2\pi\alpha')^{\frac{1}{2}}\frac{\varphi_a(p)}{\sqrt{2}}\psi^a(y)e^{-\phi(y)}e^{i\sqrt{2\pi\alpha'}p\cdot X(y)},
\end{aligned}
\tag{4}
$$

while in the 0 picture, they are given by

$$
\begin{aligned}
V_A^{(0)}(y;p) &= 2i(2\pi\alpha')^{\frac{1}{2}}A_\mu(p)\left(\partial X^\mu(y) + i(2\pi\alpha')^{\frac{1}{2}}p\cdot\psi\psi^\mu(y)\right)e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}, \\
V_\varphi^{(0)}(y;p) &= 2i(2\pi\alpha')^{\frac{1}{2}}\varphi_a(p)\left(\partial X^a(y) + i(2\pi\alpha')^{\frac{1}{2}}p\cdot\psi\psi^a(y)\right)e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}.
\end{aligned}
\tag{5}
$$

The gaugino vertex operators in the $(-1/2)$ picture are

$$
\begin{aligned}
V_\Lambda^{(-1/2)}(y;p) &= (2\pi\alpha')^{\frac{3}{4}}\Lambda^{\alpha A}(p)S_\alpha(y)S_A(y)e^{-\frac{1}{2}\phi(y)}e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}, \\
V_{\overline{\Lambda}}^{(-1/2)}(y;p) &= (2\pi\alpha')^{\frac{3}{4}}\overline{\Lambda}_{\dot{\alpha}A}(p)S^{\dot{\alpha}}(y)S^A(y)e^{-\frac{1}{2}\phi(y)}e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}.
\end{aligned}
\tag{6}
$$

We adapt dimensionless four-momentum $\sqrt{2\pi\alpha'}p$ to ensure that the momentum representation of a field have same dimension of space-time field.

2.2 R-R vertex operator

The vertex operators for massless states in the R-R sector of type IIB superstrings are constructed from the tensor product of spin fields $(S^\alpha S^A, S_{\dot{\alpha}}S_A)$ and $(\tilde{S}^\beta\tilde{S}^B, \tilde{S}_{\dot{\beta}}\tilde{S}_B)$. We will study the effect of constant R-R background, which is described by the closed string vertex operator

$$
V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z}) = (2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{\alpha\beta AB}S_\alpha S_A e^{-\frac{1}{2}\phi}(z)\tilde{S}_\beta\tilde{S}_B e^{-\frac{1}{2}\tilde{\phi}}(\bar{z}). \tag{7}
$$

We note that general massless closed string states in the R-R sector contain the field strength of types $\mathcal{F}^\alpha{}_{\dot{\alpha}}{}^A{}_B$, $\mathcal{F}_{\dot{\alpha}}{}^\alpha{}_A{}^B$, and $\mathcal{F}_{\dot{\alpha}\dot{\beta}AB}$. Since the vertex operator (7) provides a generalization of the deformations of $\mathcal{N}=2$ non(anti)commutative superspace [11], we will consider this type of deformation in the present work.

As in the $\mathcal{N}=2$ case [11], the field strength is decomposed as

$$\mathcal{F}^{\alpha\beta AB}=\mathcal{F}^{(\alpha\beta)(AB)}+\mathcal{F}^{(\alpha\beta)[AB]}+\mathcal{F}^{[\alpha\beta](AB)}+\mathcal{F}^{[\alpha\beta][AB]}. \tag{8}$$

Here the parenthesis (AB) represents symmetrization of indices A and B. $[AB]$ represents anti-symmetrization. We call these backgrounds (S,S), (S,A), (A,S), (A,A)-type deformations respectively. We now examine the correspondence between the field strength $\mathcal{F}^{\alpha\beta AB}$ and the p-form R-R field strengths in type IIB superstrings.

For four-dimensional sector, the tensor $f^{\alpha\beta}$ can be decomposed into the singlet and the self-dual tensor parts:

$$f^{\alpha\beta}=\epsilon^{\alpha\beta}f+(\sigma^{\mu\nu})^{\alpha\beta}f_{\mu\nu}. \tag{9}$$

The first term corresponds to antisymmetric part and the second to the symmetric part.

For six-dimensional sector, the spinor indices are labeled by the fundamental representation $\underline{\mathbf{4}}$ of $SU(4)$. The tensor product $\underline{\mathbf{4}}\otimes\underline{\mathbf{4}}$ can be decomposed into $\underline{\mathbf{6}}\oplus\underline{\mathbf{10}}$, which corresponds to the vector or the self-dual 3-form representation, respectively. In fact, the tensor g^{AB} is expressed as

$$g^{AB}=(\Sigma^a)^{AB}g_a+(\Sigma^{abc})^{AB}g_{abc}. \tag{10}$$

Here we define a matrix which is totally antisymmetric with respect to the space indices of abc,

$$(\Sigma^{abc})^{AB}\equiv(\Sigma^{[a}\bar{\Sigma}^{b}\Sigma^{c]})^{AB}. \tag{11}$$

The first term in (10) corresponds to the antisymmetric part and the second to the symmetric part. The matrix Σ^{abc} is self-dual:

$$(\Sigma^{abc})^{AB}=\frac{i}{3!}\epsilon^{abcdef}(\Sigma_{def})^{AB}, \tag{12}$$

and consequently the three-form g_{abc} also satisfies the self-dual condition,

$$g_{abc}=\frac{i}{3!}\epsilon_{abcdef}g^{def}. \tag{13}$$

From the decompositions (9) and (10), we find that the $\mathcal{F}^{\alpha\beta AB}$ can be decomposed into

$$\begin{aligned}\mathcal{F}^{\alpha\beta AB} &= \left(\epsilon^{\alpha\beta}f+(\sigma^{\mu\nu})^{\alpha\beta}f_{\mu\nu}\right)\times\left((\Sigma^a)^{AB}\,g_a+(\Sigma^{abc})^{AB}\,g_{abc}\right)\\ &= fg_a\epsilon^{\alpha\beta}(\Sigma^a)^{AB}+fg_{abc}\epsilon^{\alpha\beta}(\Sigma^{abc})^{AB}+f_{\mu\nu}g_a(\sigma^{\mu\nu})^{\alpha\beta}(\Sigma^a)^{AB}\\ &\quad +f_{\mu\nu}g_{abc}(\sigma^{\mu\nu})^{\alpha\beta}(\Sigma^{abc})^{AB},\end{aligned} \tag{14}$$

which corresponds to

$$\mathcal{F}^{\alpha\beta AB} \sim (\text{R-R 1-form}) \oplus (\text{R-R 3-form}) \oplus (\text{R-R 3-form}) \oplus (\text{R-R 5-form}). \tag{15}$$

The decomposition (14) shows that the (A,A) deformation corresponds to the R-R 1-form, the (A,S) and (S,A) deformations to the R-R 3-forms, and the (S,S) deformation to the R-R self-dual 5-form. In fact, if we identify the self-dual five-form field strength F^{mnpqr} as

$$F^{\mu\nu abc} = f_{\mu\nu} g_{abc}, \tag{16}$$

then it satisfies the self-dual condition in the 10-dimensional space,

$$F^{\mu\nu abc} = \frac{i}{2!3!} \epsilon^{\mu\nu abc\rho\sigma def} F_{\rho\sigma def}. \tag{17}$$

We note that the similar decomposition holds in the case of the deformation of $\mathcal{N} = 2$ super Yang-Mills theory [11], which is constructed from the type IIB superstrings compactified on $\mathbf{C} \times \mathbf{C}^2/\mathbf{Z}^2$.

2.3 Disk amplitudes and auxiliary field method

The action of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is obtained by evaluating correlation functions of the vertex operators given in equations (4), (5), (6). Let us consider disk amplitudes with boundary attached on the D3-brane world volume. The disk is realized as the upper half of complex plane. The boundary condition of the spin field [15] is

$$S_\alpha S_A(z) = \tilde{S}_\alpha \tilde{S}_A(\bar{z})\Big|_{z=\bar{z}}. \tag{18}$$

The disk amplitudes can be calculated by replacing $\tilde{S}_\alpha \tilde{S}_A(\bar{z})$ by $S_\alpha S_A(\bar{z})$ in the correlator. The $n + 2n_{\mathcal{F}}$-point disk amplitude for n vertex operators $V_{X_i}^{(q_i)}(y_i)$ and $n_{\mathcal{F}}$ R-R vertex operators $V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z_j, \bar{z}_j)$ is given by

$$\langle\langle V_{X_1}^{(q_1)} \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})} \cdots \rangle\rangle = C_{D_2} \int \frac{\prod_{i=1}^{n} dy_i \prod_{j=1}^{n_{\mathcal{F}}} dz_j d\bar{z}_j}{dV_{CKG}} \langle V_{X_1}^{(q_1)}(y_1) \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z_1, \bar{z}_1) \cdots \rangle. \tag{19}$$

Here C_{D_2} is the disk normalization factor [18]:

$$C_{D_2} = \frac{1}{2\pi^2(\alpha')^2} \frac{1}{k g_{\text{YM}}^2} \tag{20}$$

and $g_{\rm YM}$ is the gauge coupling constant. k is a normalization constant of $U(N)$ generators T^a: $\mathrm{Tr}(T^aT^b)=k\delta^{ab}$. dV_{CKG} is an $SL(2,\mathbf{R})$-invariant volume factor to fix three positions x_1, x_2 and x_3 among y_i, z_j,and $\bar{z}_j$'s:

$$dV_{CKG}=\frac{dx_1dx_2dx_3}{(x_1-x_2)(x_2-x_3)(x_3-x_1)}. \tag{21}$$

The open string amplitudes in the zero slope limit shows that the effective action on the D3-branes is that of $\mathcal{N}=4$ super Yang-Mills theory:

$$\begin{aligned}\mathcal{L}_{\rm SYM}^{\mathcal{N}=4} &= \frac{1}{k}\frac{1}{g_{\rm YM}^2}\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}\left(F_{\mu\nu}+\tilde{F}_{\mu\nu}\right)-i\Lambda^{\alpha A}(\sigma^\mu)_{\alpha\dot{\beta}}D_\mu\overline{\Lambda}^{\dot{\beta}}{}_A-\frac{1}{2}\left(D_\mu\varphi_a\right)^2\right.\\ &\left.+\frac{1}{2}\left(\Sigma^a\right)^{AB}\overline{\Lambda}_{\dot{\alpha}A}[\varphi_a,\overline{\Lambda}^{\dot{\alpha}}{}_B]+\frac{1}{2}\left(\overline{\Sigma}^a\right)_{AB}\Lambda^{\alpha A}[\varphi_a,\Lambda^B_\alpha]+\frac{1}{4}[\varphi_a,\varphi_b]^2\right],\end{aligned} \tag{22}$$

where

$$\begin{aligned}F_{\mu\nu} &= \partial_\mu A_\nu-\partial_\nu A_\mu+i[A_\mu,A_\nu],\\ D_\mu\varphi_a &= \partial_\mu\varphi_a+i[A_\mu,\varphi_a],\end{aligned} \tag{23}$$

and $A_\mu=A^a_\mu T^a$ etc. $\tilde{F}_{\mu\nu}$ is the dual of $F_{\mu\nu}$.

We use the auxiliary field method [5, 11] to simplify the string amplitudes including contact terms. We introduce the auxiliary fields $H_{\mu\nu}$, $H_{\mu a}$ and H_{ab} and rewrite the action (22) into the form

$$\begin{aligned}\mathcal{L}_{\rm SYM} &= -\frac{1}{g_{\rm YM}^2}\frac{1}{k}\mathrm{Tr}\left[\frac{1}{2}(\partial_\mu A_\nu-\partial_\nu A_\mu)\partial^\mu A^\nu+i\partial_\mu A_\nu[A^\mu,A^\nu]+\frac{1}{2}H_cH^c+\frac{1}{2}H_c\eta^c_{\mu\nu}[A^\mu,A^\nu]\right]\\ &-\frac{1}{g_{\rm YM}^2}\frac{1}{k}\mathrm{Tr}\left[\frac{1}{2}H_{ab}H_{ab}+\frac{1}{\sqrt{2}}H_{ab}[\varphi_a,\varphi_b]\right]\\ &-\frac{1}{g_{\rm YM}^2}\frac{1}{k}\mathrm{Tr}\left[\frac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi^a+i\partial_\mu\varphi_a[A^\mu,\varphi_a]+\frac{1}{2}H_{\mu a}H^{a\mu}+H_{\mu a}[A^\mu,\varphi_a]\right]\\ &-\frac{1}{g_{\rm YM}^2}\frac{1}{k}\mathrm{Tr}\left[i\Lambda^A\sigma^\mu D_\mu\bar{\Lambda}_A-\frac{1}{2}\left(\Sigma^a\right)^{AB}\overline{\Lambda}_{\dot{\alpha}A}[\varphi_a,\overline{\Lambda}^{\dot{\alpha}}{}_B]-\frac{1}{2}\left(\overline{\Sigma}^a\right)_{AB}\Lambda^{\alpha A}[\varphi_a,\Lambda^B_\alpha]\right].\end{aligned} \tag{24}$$

All quartic interactions in (22) are replaced by cubic ones. The vertex operators for auxiliary fields are given by

$$\begin{aligned}V^{(0)}_{H_{AA}}(y) &= \frac{1}{2}(2\pi\alpha')H_{\mu\nu}(p)\psi^\nu\psi^\mu e^{i\sqrt{2\pi\alpha'}p\cdot X}(y),\\ V^{(0)}_{H_{A\varphi}}(y;p) &= 2(2\pi\alpha')H_{\mu a}(p)\psi^\mu\psi^a(y)e^{i\sqrt{2\pi\alpha'}p\cdot X(y)},\\ V^{(0)}_{H_{\varphi\varphi}}(y;p) &= -\frac{1}{\sqrt{2}}(2\pi\alpha')H_{ab}(p)\psi^a\psi^b(y)e^{i\sqrt{2\pi\alpha'}p\cdot X(y)}.\end{aligned} \tag{25}$$

3 Disk amplitudes in the constant graviphoton background

In this section we calculate disk amplitudes including one graviphoton vertex operator in the zero-slope limit and study the deformed $\mathcal{N}=4$ super Yang-Mills action at the order $\mathcal{O}(\mathcal{F})$.

As in the $\mathcal{N}=1$ [5] and $\mathcal{N}=2$ [19, 11] cases, the deformed action depends on the scaling condition for the graviphoton field strength. In this paper we fix the zero-slope scaling of R-R field strength as

$$(2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{\alpha\beta AB}\equiv C^{\alpha\beta AB}=\text{fixed}. \tag{26}$$

In this scaling, the parameter $C^{\alpha\beta AB}$ has mass dimension -1, which is the same dimension as the deformation parameters in non(anti)commutative superspace. We will also focus on the (S,S)-type background $\mathcal{F}^{(\alpha\beta)(AB)}$, which corresponds to the self-dual R-R 5-form background and is expected to give a generalization of non-singlet deformation of $\mathcal{N}=2$ superspace [11].

When the R-R vertex operator (7) is inserted in the disk, the charge conservation for internal spin fields restricts possible insertions of the open string vertex operators. In fact, the operators of types $\overline{\Lambda}\overline{\Lambda}$, $\Lambda\overline{\Lambda}\varphi$ and $\varphi\varphi\varphi$ cancel the internal charge of the R-R vertex operator. In the zero slope limit with the scaling condition (26), we find that the following amplitudes become nonzero in the (S,S)-type background:

$$\langle\!\langle V_A^{(0)}V_{\overline{\Lambda}}^{(-1/2)}V_{\overline{\Lambda}}^{(-1/2)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle+\langle\!\langle V_{H_{AA}}^{(0)}V_{\overline{\Lambda}}^{(-1/2)}V_{\overline{\Lambda}}^{(-1/2)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle, \tag{27}$$

$$\langle\!\langle V_{\Lambda}^{(-1/2)}V_{\overline{\Lambda}}^{(-1/2)}V_{\varphi}^{(0)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle+\langle\!\langle V_{\Lambda}^{(-1/2)}V_{\overline{\Lambda}}^{(-1/2)}V_{H_{\varphi\varphi}}^{(0)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle, \tag{28}$$

$$\begin{aligned}&\langle\!\langle V_{\varphi}^{(0)}V_{\varphi}^{(0)}V_{\varphi}^{(-1)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle+\langle\!\langle V_{H_{A\varphi}}^{(0)}V_{\varphi}^{(0)}V_{\varphi}^{(-1)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle\\&\quad+\langle\!\langle V_{H_{A\varphi}}^{(0)}V_{H_{A\varphi}}^{(0)}V_{\varphi}^{(-1)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle,\end{aligned} \tag{29}$$

$$\langle\!\langle V_A^{(0)}V_{H_{\varphi\varphi}}^{(0)}V_{\varphi}^{(-1)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle+\langle\!\langle V_{H_{A\varphi}}^{(0)}V_{H_{\varphi\varphi}}^{(0)}V_{\varphi}^{(-1)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle, \tag{30}$$

$$\langle\!\langle V_{H_{\varphi\varphi}}^{(0)}V_{\Lambda}^{(-1/2)}V_{\Lambda}^{(-1/2)}V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle. \tag{31}$$

As in the $\mathcal{N}=1$ [5] and $\mathcal{N}=2$ [11] cases, gauge invariance in the effective action is ensured by the fact that the derivative $\partial_\mu A_\nu$ or $\partial_\mu\varphi$ appears together with auxiliary fields

$H_{\mu\nu}$ and $H_{A\varphi}$, respectively. The derivative terms turn out to be covariant derivatives after integrating out auxiliary fields. Appropriate weight factors of the amplitudes must be taken into account to keep the gauge invariance of the results. We now compute the amplitudes (27)–(30) explicitly.

$\bullet\langle\langle V_A V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}}\rangle\rangle + \langle\langle V_{H_{AA}} V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}}\rangle\rangle$

The first term of the amplitudes (27) is given by

$$\begin{aligned}
&\langle\langle V_A^{(0)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{\overline{\Lambda}}^{(-1/2)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\rangle \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2i)(2\pi\alpha')^3\mathrm{Tr}\left[A_\mu(p_1)\overline{\Lambda}_{\dot{\alpha}C}(p_2)\overline{\Lambda}_{\dot{\beta}D}(p_3)\right]\mathcal{F}^{(\alpha\beta)(AB)} \\
&\quad\times\int\frac{\prod_j dy_j dz d\bar{z}}{dV_{\mathrm{CKG}}}\ \langle e^{-\frac{1}{2}\phi(y_1)}e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle\langle S^C(y_1)S^D(y_2)S_A(z)S_B(\bar{z})\rangle \\
&\quad\times\left\langle\left(\partial X^\mu(y_1)+i(2\pi\alpha')^{\frac{1}{2}}p_{1\nu}\psi^\nu\psi^\mu(y_1)\right)S^{\dot{\alpha}}(y_2)S^{\dot{\beta}}(y_3)S_\alpha(z)S_\beta(\bar{z})\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle.
\end{aligned} \tag{32}$$

We note that ∂X^μ in the last correlator of (32) does not contribute to the amplitude because of symmetric property of $\mathcal{F}^{(\alpha\beta)(AB)}$. The correlation functions are calculated by using bosonization formulas summarized in Appendix A. We then perform the world-sheet integral of the form

$$\int_{-\infty}^{\infty}dy_2\int_{-\infty}^{y_2}dy_3\ \frac{(z-\bar{z})^2}{(y_2-z)(y_2-\bar{z})(y_3-z)(y_3-\bar{z})}=(2i)^2\frac{\pi^2}{2} \tag{33}$$

which is done by fixing the world-sheet coordinates to $z=i, \bar{z}=-i, y_1\to\infty$. The amplitude becomes

$$\begin{aligned}
&\langle\langle V_A^{(0)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{\overline{\Lambda}}^{(-1/2)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\rangle \\
&= -\frac{4\pi^2 i}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}ip_{1[\mu}A_{\nu]}(p_1)\overline{\Lambda}_{\dot{\alpha}A}(p_2)\overline{\Lambda}^{\dot{\alpha}}{}_B(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}.
\end{aligned} \tag{34}$$

The second term in (27) can be evaluated in the same way. The result is

$$\begin{aligned}
&\langle\langle V_H^{(0)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{\overline{\Lambda}}^{(-1/2)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\rangle \\
&\quad= -\frac{1}{2i}\frac{1}{2}\frac{8\pi^2 i}{kg_{\mathrm{YM}}^2}\mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}H_{\mu\nu}(p_1)\overline{\Lambda}_{\dot{\alpha}A}(p_2)\overline{\Lambda}^{\dot{\alpha}}{}_B(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}.
\end{aligned} \tag{35}$$

We need to add another color order contribution, which actually gives the same result and cancels the symmetric factor 1/2!. The interaction terms in the effective Lagrangian obtained from the amplitudes (34) and (35) are given by

$$\mathcal{L}_1 = \frac{4\pi^2 i}{k g_{\mathrm{YM}}^2} \mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}\left(\partial_{[\mu}A_{\nu]} - \frac{i}{2}H_{\mu\nu}\right)\overline{\Lambda}_{\dot{\alpha}A}\overline{\Lambda}^{\dot{\alpha}}{}_{B}\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}. \tag{36}$$

$\bullet\langle\!\langle V_\Lambda V_{\overline{\Lambda}} V_\varphi V_{\mathcal{F}}\rangle\!\rangle + \langle\!\langle V_\Lambda V_{\overline{\Lambda}} V_{H_{\varphi\varphi}} V_{\mathcal{F}}\rangle\!\rangle$

The first term in the amplitudes (28) is given by

$$\begin{aligned}
&\langle\!\langle V_\Lambda^{(-1/2)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_\varphi^{(0)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{k g_{\mathrm{YM}}^2}(2i)(2\pi\alpha')^3 \mathrm{Tr}\left[\Lambda^{\gamma C}(p_1)\overline{\Lambda}_{\dot{\beta}D}(p_2)\varphi_a(p_3)\right]\mathcal{F}^{(\alpha\beta)(AB)} \\
&\quad\times\int\frac{\prod_j dy_j dz d\bar{z}}{dV_{\mathrm{CKG}}}\ \langle e^{-\frac{1}{2}\phi(y_1)}e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle \\
&\quad\times\Big\langle\ S_\gamma S_C(y_1)S^{\dot{\alpha}}S^D(y_2)\left(\partial X^a(y_3) + i(2\pi\alpha')^{\frac{1}{2}}p_{3\mu}\psi^\mu\psi^a(y_3)\right) \\
&\quad\times S_\alpha S_A(z)S_\beta S_B(\bar{z})\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\Big\rangle.
\end{aligned} \tag{37}$$

Here ∂X^a does not contribute to the amplitude for the (S,S)-type background. Using the formula for the five point function of spin fields in Appendix A, we obtain

$$\begin{aligned}
&\langle\!\langle V_\Lambda^{(-1/2)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_\varphi^{(0)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{4\pi^2 i}{k g_{\mathrm{YM}}^2}\mathrm{Tr}\left[\Lambda_\alpha{}^C(p_1)(\overline{\Sigma}^a)_{AC}\overline{\Lambda}_{\dot{\alpha}B}(p_2)(\sigma^\mu)_\beta{}^{\dot{\alpha}}ip_{3\mu}\varphi_a(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}.
\end{aligned} \tag{38}$$

The amplitude which includes the auxiliary field $H_{\mu a}$ is given by

$$\begin{aligned}
&\langle\!\langle V_\Lambda^{(-1/2)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{H_{A\varphi}}^{(0)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{2}{2i}\frac{4\pi^2 i}{k g_{\mathrm{YM}}^2}\mathrm{Tr}\left[\Lambda_\alpha{}^C(p_1)(\overline{\Sigma}^a)_{AC}\overline{\Lambda}_{\dot{\alpha}B}(p_2)(\sigma^\mu)_\beta{}^{\dot{\alpha}}H_{\mu a}(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}.
\end{aligned} \tag{39}$$

Another color order contribution needs to be added. These amplitudes are obtained from the interaction

$$\mathcal{L}_2 = \frac{4\pi^2 i}{k g_{\mathrm{YM}}^2}\mathrm{Tr}\left[\left\{(\sigma^\mu)_\alpha{}^{\dot{\alpha}}\left(\partial_\mu\varphi_a - iH_{\mu a}\right), \overline{\Lambda}_{\dot{\alpha}A}\right\}(\overline{\Sigma}^a)_{BC}\Lambda_\beta{}^C\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}. \tag{40}$$

•$\langle\!\langle \varphi\varphi\varphi\mathcal{F}\rangle\!\rangle + \langle\!\langle H_{A\varphi}\varphi\varphi\mathcal{F}\rangle\!\rangle + \langle\!\langle H_{A\varphi}H_{A\varphi}\varphi\mathcal{F}\rangle\!\rangle$

The first term in (29) is given by

$$
\begin{aligned}
&\langle\!\langle V_\varphi^{(0)}(p_1)V_\varphi^{(0)}(p_2)V_\varphi^{(-1)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\rm YM}^2}(2i)^2\frac{1}{\sqrt{2}}(2\pi\alpha')^{\frac{5}{2}}\,{\rm Tr}\left[\varphi_a(p_1)\varphi_b(p_2)\varphi_c(p_3)\right]\mathcal{F}^{(\alpha\beta)(AB)} \\
&\quad\times\int\frac{\prod_j dy_j dz d\bar{z}}{dV_{\rm CKG}}\ \langle e^{-\phi(y_3)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle \\
&\quad\times\Big\langle\Big(\partial X^a(y_1)+i(2\pi\alpha')^{\frac{1}{2}}p_{1\mu}\psi^\mu\psi^a(y_1)\Big)\Big(\partial X^b(y_2)+i(2\pi\alpha')^{\frac{1}{2}}p_{2\nu}\psi^\nu\psi^b(y_2)\Big) \\
&\quad\times\psi^c(y_3)S_\alpha(z)S_\beta(\bar{z})S_A(z)S_\beta(\bar{z})S_B(\bar{z})\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\Big\rangle.
\end{aligned}
\tag{41}
$$

In the above amplitudes the term containing $\partial X^a\partial X^b$ gives the contribution $\langle S_\alpha S_\beta\rangle\sim\varepsilon_{\alpha\beta}$, which becomes zero after the contraction with the (S,S)-type background. The terms containing the single ∂X do not contribute to the amplitude due to $\langle S_\alpha\psi^\mu S_\beta\rangle = 0$. We obtain

$$
\begin{aligned}
&\langle\!\langle V_\varphi^{(0)}(p_1)V_\varphi^{(0)}(p_2)V_\varphi^{(-1)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= -\frac{4\pi^2}{kg_{\rm YM}^2}{\rm Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}ip_{1\mu}\varphi_a(p_1)ip_{2\nu}\varphi_b(p_2)\varphi_c(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}
\end{aligned}
\tag{42}
$$

The amplitudes including auxiliary fields can be calculated in a similar way. Multiplying appropriate weight and symmetric factors, the interaction terms in the Lagrangian are shown to become

$$
\begin{aligned}
\mathcal{L}_3 \;=\; & \frac{1}{3}\frac{4\pi^2}{kg_{\rm YM}^2}{\rm Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}\partial_\mu\varphi_a\partial_\nu\varphi_b\varphi_c\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)} \\
&+\frac{1}{3}\frac{2}{2i}\frac{4\pi^2}{kg_{\rm YM}^2}{\rm Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}\{H_{\mu a},\partial_\nu\varphi_b\}\varphi_c\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)} \\
&+\frac{1}{3}\frac{2^2}{(2i)^2}\frac{4\pi^2}{kg_{\rm YM}^2}{\rm Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}H_{\mu a}H_{\nu b}\varphi_c\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}.
\end{aligned}
\tag{43}
$$

•$\langle\!\langle A_\mu H_{ab}\varphi\mathcal{F}\rangle\!\rangle + \langle\!\langle H_{\mu\nu}H_{ab}\varphi\mathcal{F}\rangle\!\rangle$

Now we compute the amplitude (30). The first term in (30) is given by

$$\begin{aligned}
&\langle\!\langle V_A^{(0)}(p_1)V_{H_{\varphi\varphi}}^{(0)}(p_2)V_\varphi^{(-1)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\rm YM}^2}(2i)\left(-\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}(2\pi\alpha')^3\mathrm{Tr}\left[A_\mu(p_1)H_{ab}(p_2)\varphi_c(p_3)\right]\mathcal{F}^{(\alpha\beta)(AB)} \\
&\quad\times\int\frac{\prod_j dy_j dz d\bar{z}}{dV_{\rm CKG}}\ \langle e^{-\phi(y_3)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle\langle\psi^a\psi^b(y_2)\psi^c(y_3)S_A(z)S_B(\bar{z})\rangle \\
&\quad\times\left\langle\left(\partial X^\mu(y_1)+i(2\pi\alpha')^{\frac{1}{2}}p_{1\nu}\psi^\nu\psi^\mu(y_1)\right)S_\alpha(z)S_\beta(\bar{z})\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle. \qquad (44)
\end{aligned}$$

The term including ∂X does not contribute to the amplitude for the (S,S)-type background again. Evaluating the correlation functions, we obtain

$$\begin{aligned}
&\langle\!\langle V_A^{(0)}(p_1)V_{H_{\varphi\varphi}}^{(0)}(p_2)V_\varphi^{(-1)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{i\sqrt{2}\pi^2}{kg_{\rm YM}^2}\mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}ip_{1\mu}A_\nu(p_1)H_{ab}(p_2)\varphi_c(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}. \qquad (45)
\end{aligned}$$

Taking into account other color ordered contributions and adding the second term in (30), the interaction terms become

$$\mathcal{L}_4 = -\frac{\sqrt{2}\pi^2 i}{kg_{\rm YM}^2}\mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}\left(\partial_{[\mu}A_{\nu]}-\frac{i}{2}H_{\mu\nu}\right)\{H_{ab},\varphi_c\}\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}. \qquad (46)$$

•$\langle\!\langle H_{\varphi\varphi}\Lambda\Lambda\mathcal{F}\rangle\!\rangle$

This amplitude is given by

$$\begin{aligned}
&\langle\!\langle V_{\varphi\varphi}^{(0)}(p_1)V_\Lambda^{(-1/2)}(p_2)V_\Lambda^{(-1/2)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\rm YM}^2}(2\pi\alpha')^{2+\frac{3}{2}}\left(-\frac{1}{\sqrt{2}}\right)\mathrm{Tr}\left[H_{ab}(p_1)\Lambda^{\gamma C}(p_2)\Lambda^{\delta D}(p_3)\right]\mathcal{F}^{(\alpha\beta)(AB)} \\
&\times\int\frac{\prod_j dy_j dz d\bar{z}}{dV_{\rm CKG}}\langle e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(y_3)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle \\
&\times\langle\psi^a\psi^b(y_1)S_C(y_2)S_D(y_3)S_A(z)S_B(\bar{z})\rangle\langle S_\gamma(y_2)S_\delta(y_3)S_\alpha(z)S_\beta(\bar{z})\rangle. \qquad (47)
\end{aligned}$$

Using the formula (75) in the appendix A, we get

$$
\begin{aligned}
&\langle\!\langle V^{(0)}_{\varphi\varphi}(p_1)V^{(-1/2)}_{\Lambda}(p_2)V^{(-1/2)}_{\Lambda}(p_3)V^{(-1/2,-1/2)}_{\mathcal{F}}\rangle\!\rangle \\
&\quad = \frac{2\sqrt{2}\pi^2}{kg^2_{\rm YM}}\mathrm{Tr}\left[(\overline{\Sigma}^{ab})_A{}^{A'}\varepsilon_{CDA'B}H_{ab}(p_1)\Lambda_\alpha{}^C(p_2)\Lambda_\beta{}^D(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}. \quad (48)
\end{aligned}
$$

Adding the color ordered amplitude and considering weight and phase factors, we find that the interaction term is

$$
\mathcal{L}_5 = -\frac{2\sqrt{2}\pi^2 i}{kg^2_{\rm YM}}\mathrm{Tr}\left[(\overline{\Sigma}^{ab})_A{}^{A'}\varepsilon_{CDA'B}H_{ab}\Lambda_\alpha{}^C\Lambda_\beta{}^D\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}. \quad (49)
$$

To summarize, the first order correction to the $\mathcal{N}=4$ super Yang-Mills action from the (S,S)-type graviphoton background is

$$
\mathcal{L}^{(1)}_{\rm (S,S)} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + a_1\mathcal{L}_4 + a_2\mathcal{L}_5. \quad (50)
$$

Here we have introduced additional weight factors a_1 and a_2, which should be determined by higher point disk amplitudes without auxiliary fields. In this paper we will determine these weight factors such that the deformed Lagrangian is consistent with the one defined on $\mathcal{N}=1/2$ superspace.

By integrating out the auxiliary fields and defining the deformation parameter C by $C^{(\alpha\beta)(AB)} \equiv -8\pi^2(2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(AB)}$, we find the deformed Lagrangian is expressed as

$$
\mathcal{L} = \mathcal{L}^{\mathcal{N}=4}_{\rm SYM} + \mathcal{L}^{(1)}_{\rm (S,S)} + \mathcal{O}(C^2) \quad (51)
$$

where $\mathcal{L}^{\mathcal{N}=4}_{\rm SYM}$ is the ordinary $\mathcal{N}=4$ super Yang-Mills action (22) and

$$
\begin{aligned}
\mathcal{L}^{(1)}_{\rm (S,S)} \;=\; & -\frac{i}{2}\frac{1}{kg^2_{\rm YM}}\mathrm{Tr}\left[F_{\mu\nu}\overline{\Lambda}_{\dot\alpha A}\overline{\Lambda}^{\dot\alpha}{}_B\right]C^{\mu\nu(AB)} \\
& -\frac{i}{2}\frac{1}{kg^2_{\rm YM}}\mathrm{Tr}\left[\left\{D_\mu\varphi_a,(\sigma^\mu)_{\alpha\dot\alpha}\overline{\Lambda}^{\dot\alpha}{}_A\right\}(\overline{\Sigma}^a)_{BC}\Lambda_\beta{}^C\right]C^{(\alpha\beta)(AB)} \\
& -\frac{1}{6}\frac{1}{kg^2_{\rm YM}}\mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}D_\mu\varphi_a D_\nu\varphi_b\varphi_c\right]C^{(\alpha\beta)(AB)} \\
& -\frac{i}{3}\frac{a_1}{kg^2_{\rm YM}}\mathrm{Tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}F_{\mu\nu}\varphi_a\varphi_b\varphi_c\right]C^{(\alpha\beta)(AB)} \\
& -\frac{i}{4}\frac{a_2}{kg^2_{\rm YM}}\mathrm{Tr}\left[(\overline{\Sigma}^{ab})_A{}^{A'}\varepsilon_{A'BCD}\varphi_a\varphi_b\Lambda_\alpha{}^C\Lambda_\beta{}^D\right]C^{(\alpha\beta)(AB)}. \quad (52)
\end{aligned}
$$

Here $C^{\mu\nu(AB)}$ is defined by $C^{\mu\nu(AB)} = C^{(\alpha\beta)(AB)}\varepsilon_{\beta\gamma}(\sigma^{\mu\nu})_\alpha{}^\gamma$. We note that the bosonic terms in (52) gives the ones obtained from the Chern-Simons term [12] in the R-R potential, which was also observed from the $\mathcal{N}=4$ super Yang-Mills theory on non(anti)commutative $\mathcal{N}=1$ superspace [14] (see also appendix B). The reduction to deformed $\mathcal{N}=1$ superspace is done by restriction of the deformation parameter $C^{(\alpha\beta)(AB)}$ to $C^{(\alpha\beta)(11)}$. The deformed Lagrangian (52) agrees with the non(anti)commutative one in [14] if we choose the weight factors to be $a_1=-\frac{1}{2}, a_2=-4i$.

4 Vacuum structure of deformed $\mathcal{N}=4$ theory

In this section we study the vacuum structure of deformed $\mathcal{N}=4$ SYM theory based on the Lagrangian (51). For simplicity, we take $\Lambda=\overline{\Lambda}=0$, and consider φ_a as constants. We also assume that only $U(1)$ part of gauge field strength $F^{U(1)}_{\mu\nu}$ is non-vanishing constant. In this case, the Lagrangian becomes

$$\mathcal{L}_{\rm scalar}=\frac{1}{kg^2_{\rm YM}}{\rm Tr}\left[\frac{1}{4}[\varphi_a,\varphi_b]^2+\frac{i}{6}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}F^{U(1)}_{\mu\nu}C^{\mu\nu(AB)}\varphi_a\varphi_b\varphi_c\right]. \tag{53}$$

The equation of motion is given by

$$[\varphi_b,[\varphi_a,\varphi_b]]+\frac{i}{4}(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}F^{U(1)}_{\mu\nu}C^{\mu\nu(AB)}[\varphi_b,\varphi_c]=0. \tag{54}$$

We want to find the solution with the ansatz

$$\begin{aligned}&[\varphi_{\hat a},\varphi_{\hat b}]=i\kappa\varepsilon_{\hat a\hat b\hat c}\varphi_{\hat c},\quad (\hat a,\hat b,\hat c=1,2,3),\\ &\varphi_{\hat i}=0\quad (\hat i=4,5,6).\end{aligned} \tag{55}$$

Taking the contraction with $C^{(AB)}$, totally antisymmetric part of $(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}$ remains. We find that the equation (54) reduces to

$$[\varphi_{\hat b},[\varphi_{\hat a},\varphi_{\hat b}]]+\frac{i}{4}\varepsilon_{\hat a\hat b\hat c}(F\cdot C)[\varphi_{\hat b},\varphi_{\hat c}]=0 \tag{56}$$

where $(\overline{\Sigma}^{\hat a}\Sigma^{\hat b}\overline{\Sigma}^{\hat c})_{AB}C^{\mu\nu(AB)}$ is written as $\varepsilon^{\hat a\hat b\hat c}M_{AB}C^{\mu\nu(AB)}$ for a symmetric matrix M_{AB} and $F\cdot C\equiv F^{U(1)}_{\mu\nu}C^{\mu\nu(AB)}M_{AB}$. Applying the ansatz (55) we find (54) can be rewritten as

$$\left(\kappa^2+\frac{1}{4}(F\cdot C)\kappa\right)\varepsilon_{\hat a\hat b\hat c}\varepsilon_{\hat b\hat c\hat d}\varphi_{\hat d}=0. \tag{57}$$

Thus the constant κ should be

$$\text{(i)}\qquad \kappa=0, \tag{58}$$

$$\text{(ii)}\qquad \kappa=-\frac{1}{4}(F\cdot C). \tag{59}$$

In the case (i) this gives the ordinary commutative configuration of D3-branes. However, in the case (ii), due to the non-zero R-R background C, we have the fuzzy two-sphere configuration $[\varphi_{\hat{a}}, \varphi_{\hat{b}}] = i\kappa\varepsilon_{\hat{a}\hat{b}\hat{c}}\varphi_{\hat{c}}$. Here we regard $\varphi_{\hat{a}}$ as the generators of $SU(2)$ subalgebra embedded in the N-dimensional matrix representation of the gauge group $U(N)$, which are normalized as $\varphi^{\hat{a}}\varphi^{\hat{a}} = t\mathbf{1}_{N\times N}$. The radius of the fuzzy two-sphere is given by

$$R^2 \equiv \varphi_{\hat{a}}^2 = \kappa^2 t\mathbf{1}_{N\times N}. \tag{60}$$

5 Conclusions and discussion

In this paper, we have investigated the effects of constant self-dual R-R graviphoton 5-form background to the $U(N)$ $\mathcal{N} = 4$ super Yang-Mills theory defined on D3-brane world-volume. In this paper we discussed the first order correction from the graviphoton background to the $\mathcal{N} = 4$ super Yang-Mills theory keeping $(2\pi\alpha')^{\frac{3}{2}}\mathcal{F} = C$ fixed in the zero-slope limit. This scaling gives the same dimension with the non(anti)commutativity parameter of superspace. The deformed action would be defined on the non(anti)commutative $\mathcal{N} = 4$ superspace which is characterized by the Clifford algebra $\{\theta^{\alpha A}, \theta^{\beta B}\} = C^{(\alpha\beta)(AB)}$. It would be interesting to study this deformation by using pure-spinor formalism [20] of superstring, which provides a useful method to studying higher order corrections.

By restricting the R-R 5-form field strength to the $\mathcal{N} = 1$ deformation parameter and assigning appropriate weight factors to the amplitudes, we found that the effective action agrees with the one defined on $\mathcal{N} = 1/2$ superspace [14] in our convention. We also find the fuzzy two-sphere vacuum configuration which is induced by non-zero R-R background as in [12]. In our calculation, there are no tadpole contribution nor divergent structure of the disk amplitudes which suggest the consistency of the constant R-R 5-form background with flat space-time.

We can do similar calculations for the other types of R-R background such as (S,A), (A,S), (A,A). As pointed out in [11], the (S,A) and (A,S)-type backgrounds can not be interpreted as ordinary non(anti)commutative deformation of superspace because their index structures are different. On the other hand, (A,A)-type background, which corresponds to R-R 1-form background, would provide the non(anti)commutative deformation of superspace.

Another interesting issue is to choose different scaling conditions for the R-R background in the zero-slope limit. For example, in $\mathcal{N}=2$ case, the (S,A)-type background with the scaling $(2\pi\alpha')^{\frac{1}{2}}\mathcal{F}=C$ is studied in [19]. The R-R three-form is regarded as the Ω-background, which was used for the integration over the instanton moduli space [21]. The C-deformation scaling $(2\pi\alpha')^{-\frac{1}{2}}\mathcal{F}=C$ would be also interesting [1].

Acknowledgments

We would like to thank A. Lerda for useful comments. The work of K. I. is supported in part by the Grant-in-Aid for Scientific Research No. 18540255 from Ministry of Education, Science, Culture and Sports of Japan. S. S. is supported by Yukawa Memorial Foundation.

A $\mathcal{N}=4$ effective rules

In this appendix we summarize some definitions and useful formulas which appear in this paper.

We define spin fields in six dimensions by

$$
\begin{aligned}
&S^1 = e^{\frac{1}{2}\phi_3+\frac{1}{2}\phi_4+\frac{1}{2}\phi_5}, \qquad S_1 = e^{-\frac{1}{2}\phi_3-\frac{1}{2}\phi_4-\frac{1}{2}\phi_5},\\
&S^2 = ie^{\frac{1}{2}\phi_3-\frac{1}{2}\phi_4-\frac{1}{2}\phi_5}, \qquad S_2 = ie^{-\frac{1}{2}\phi_3+\frac{1}{2}\phi_4+\frac{1}{2}\phi_5},\\
&S^3 = i^2e^{-\frac{1}{2}\phi_3+\frac{1}{2}\phi_4-\frac{1}{2}v\phi_5}, \qquad S_3 = i^2e^{\frac{1}{2}\phi_3-\frac{1}{2}\phi_4+\frac{1}{2}\phi_5},\\
&S^4 = i^3e^{-\frac{1}{2}\phi_3-\frac{1}{2}\phi_4+\frac{1}{2}\phi_5}, \qquad S_4 = i^3e^{\frac{1}{2}\phi_3+\frac{1}{2}\phi_4-\frac{1}{2}\phi_5}.
\end{aligned}
\tag{61}
$$

The correlation functions for ten-dimensional spin fields can be realized as the product of four-dimensional correlator and six-dimensional ones. Each correlator is expressed in terms of gamma matrices, which is evaluated by using the effective rules listed below.

We firstly write down the correlation functions for four-dimensional spin fields:

$$\langle S_\alpha(z)S_\beta(\bar{z})\rangle = \varepsilon_{\alpha\beta}(z-\bar{z})^{-\frac{1}{2}}, \tag{62}$$

$$\langle S^{\dot{\alpha}}(y_1)S^{\dot{\beta}}(y_2)\rangle = \varepsilon^{\dot{\alpha}\dot{\beta}}(y_1-y_2)^{-\frac{1}{2}}, \tag{63}$$

$$\langle S^{\dot{\alpha}}(y_1)S^{\dot{\beta}}(y_2)S_\alpha(z)S_\beta(\bar{z})\rangle = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta}(y_1-y_2)^{-\frac{1}{2}}(z-\bar{z})^{-\frac{1}{2}}, \tag{64}$$

$$
\begin{aligned}
\langle S^\alpha(y_1)S^\beta(y_2)S^\gamma(z)S^\delta(\bar z)\rangle &= [(y_1-y_2)(y_1-z)(y_1-\bar z)(y_2-z)(y_2-\bar z)(z-\bar z)]^{-\frac{1}{2}} \\
&\qquad \times \left[\varepsilon^{\alpha\delta}\varepsilon^{\beta\gamma}(y_1-z)(y_2-\bar z)-\varepsilon^{\alpha\gamma}\varepsilon^{\beta\delta}(y_2-z)(y_1-\bar z)\right] \\
&= [(y_1-y_2)(y_1-z)(y_1-\bar z)(y_2-z)(y_2-\bar z)(z-\bar z)]^{-\frac{1}{2}} \\
&\qquad \times \left[-\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}(y_1-\bar z)(y_2-z)+\varepsilon^{\alpha\delta}\varepsilon^{\beta\gamma}(y_1-y_2)(z-\bar z)\right].
\end{aligned}
\tag{65}
$$

The correlators including world-sheet fermions become for example

$$
\langle S^{\dot\alpha}(y_1)\psi^\mu(y_2)S_\alpha(y_3)\rangle = \frac{1}{\sqrt{2}}(\bar\sigma^\mu)^{\dot\alpha}{}_\alpha(y_1-y_2)^{-\frac{1}{2}}(y_2-y_3)^{-\frac{1}{2}}, \tag{66}
$$

$$
\begin{aligned}
&\langle \psi^\mu\psi^\nu(y_1)S^{\dot\alpha}(y_2)S^{\dot\beta}(y_3)S_\alpha(z)S_\beta(\bar z)\rangle \\
&= (y_2-y_3)^{-\frac{1}{2}}(z-\bar z)^{-\frac{1}{2}}\left[(\bar\sigma^{\mu\nu})^{\dot\alpha\dot\beta}\varepsilon_{\alpha\beta}\frac{(y_2-y_3)}{(y_1-y_2)(y_1-y_3)}+(\sigma^{\mu\nu})_{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\frac{(z-\bar z)}{(y_1-z)(y_1-\bar z)}\right].
\end{aligned}
\tag{67}
$$

Next, the correlators for six-dimensional spin fields used in this paper are

$$
\begin{aligned}
&\langle S^A(z)S_B(w)\rangle = \delta^A{}_B(z-w)^{-\frac{3}{4}}, \\
&\langle S^A(z)S^B(w)\rangle = \langle S_A(z)S_B(w)\rangle = 0,
\end{aligned}
\tag{68}
$$

$$
\langle S_A(z_1)S_B(z_2)S_C(z_3)S_D(z_4)\rangle = \frac{\epsilon_{ABCD}}{(z_1-z_2)^{\frac{1}{4}}(z_1-z_3)^{\frac{1}{4}}(z_1-z_4)^{\frac{1}{4}}(z_2-z_3)^{\frac{1}{4}}(z_2-z_4)^{\frac{1}{4}}(z_3-z_4)^{\frac{1}{4}}} \tag{69}
$$

and

$$
\begin{aligned}
&\langle S^A(y_1)S^B(y_2)S_C(y_3)S_D(y_4)\rangle \\
&\quad = (z_1-z_2)^{-\frac{1}{4}}(z_1-z_3)^{-\frac{3}{4}}(z_1-z_4)^{-\frac{3}{4}}(z_2-z_3)^{-\frac{3}{4}}(z_2-z_4)^{-\frac{3}{4}}(z_3-z_4)^{-\frac{1}{4}} \\
&\qquad \times \left[-(z_1-z_4)(z_2-z_3)\delta^A{}_C\delta^B{}_D+(z_1-z_3)(z_2-z_4)\delta^A{}_D\delta^B{}_C\right].
\end{aligned}
\tag{70}
$$

Here ϵ_{ABCD} is an anti-symmetric tensor with $\epsilon_{1234}=1$. The correlators including world-sheet fermions are

$$
\langle \psi^a(y_1)S_A(z)S_B(\bar z)\rangle = \frac{1}{\sqrt{2}}\left(\overline{\Sigma}^a\right)_{AB}(y_1-z)^{-\frac{1}{2}}(y_1-\bar z)^{-\frac{1}{2}}(z-\bar z)^{-\frac{1}{4}}, \tag{71}
$$

$$
\begin{aligned}
&\langle \psi^a\psi^b(y_1)\psi^c(y_2)S_A(z)S_B(\bar z)\rangle\\
&= +\frac{1}{\sqrt{2}}\frac{1}{y_1-y_2}\left[\delta^{ac}(\overline{\Sigma}^b)_{AB}(y_2-z)^{-\frac{1}{2}}(y_2-\bar z)^{-\frac{1}{2}}(z-\bar z)^{\frac{1}{4}}\right]\\
&\quad +\frac{1}{\sqrt{2}}\frac{1}{y_1-y_2}\left[\delta^{bc}(\overline{\Sigma}^a)_{AB}(y_2-z)^{-\frac{1}{2}}(y_2-\bar z)^{-\frac{1}{2}}(z-\bar z)^{\frac{1}{4}}\right]\\
&\quad -\frac{1}{2\sqrt{2}}\frac{1}{y_1-z}\left[(\overline{\Sigma}^{ab})_A{}^{B'}(\overline{\Sigma}^c)_{B'B}(y_2-z)^{-\frac{1}{2}}(y_2-\bar z)^{-\frac{1}{2}}(z-\bar z)^{\frac{1}{4}}\right]\\
&\quad +\frac{1}{2\sqrt{2}}\frac{1}{y_1-\bar z}\left[(\overline{\Sigma}^{ab})_B{}^{B'}(\overline{\Sigma}^c)_{AB'}(y_2-z)^{-\frac{1}{2}}(y_2-\bar z)^{-\frac{1}{2}}(z-\bar z)^{\frac{1}{4}}\right], \qquad (72)
\end{aligned}
$$

where we have defined $(\overline{\Sigma}^{ab})_A{}^B = \frac{1}{4}\Big((\overline{\Sigma}^a)_{AC}(\Sigma^b)^{CB} - (\overline{\Sigma}^b)_{AC}(\Sigma^a)^{CB}\Big)$. The following formulas are valid only when they are contracted with the (S,S)-type background $C^{(\alpha\beta)(AB)}$:

$$
\begin{aligned}
&\langle S_\gamma S_C(y_1)S^{\dot\alpha}S^D(y_2)\psi^\mu\psi^a(y_3)S_\alpha S_A(z)S_\beta S_D(\bar z)\rangle\\
&= \frac{1}{2}\varepsilon_{\gamma\beta}(\sigma^\mu)_\alpha{}^{\dot\alpha}(\overline{\Sigma}^a)_{AC}\delta^D_B(y_1-y_2)^{-\frac{3}{4}}(y_1-z)^{-\frac{3}{4}}(y_1-\bar z)^{-\frac{3}{4}}(y_1-y_2)\\
&\qquad \times(y_2-z)^{-\frac{3}{4}}(y_2-\bar z)^{-\frac{3}{4}}(y_3-z)^{-1}(y_3-\bar z)^{-1}(z-\bar z)^{-\frac{3}{4}}(z-\bar z)^2, \qquad (73)
\end{aligned}
$$

$$
\begin{aligned}
&\langle \psi^\mu\psi^a(y_1)\psi^\nu\psi^b(y_2)\psi^c(y_3)S_\alpha S_A(z)S_\beta S_B(\bar z)\rangle\\
&= -\frac{1}{4\sqrt{2}}(\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma^\nu)^{\dot\alpha}{}_\beta(\overline{\Sigma}^a\Sigma^b\overline{\Sigma}^c)_{AB}\\
&\quad \times(y_1-z)^{-1}(y_1-\bar z)^{-1}(y_2-z)^{-1}(y_2-\bar z)^{-1}(y_3-z)^{-\frac{1}{2}}(y_3-\bar z)^{-\frac{1}{2}}(z-\bar z)^{\frac{5}{4}}. \qquad (74)
\end{aligned}
$$

$$
\begin{aligned}
&\langle \psi^a\psi^b(y_1)S_C(y_2)S_D(y_3)S_A(z)S_B(\bar z)\rangle\\
&= (\overline{\Sigma}^{ab})_A{}^{A'}\varepsilon_{CDA'B}\frac{z-\bar z}{(y_1-z)(y_1-\bar z)}\left[(y_2-y_3)(y_2-z)(y_2-\bar z)(y_3-z)(y_3-\bar z)(z-\bar z)\right]^{-\frac{1}{4}}. \qquad (75)
\end{aligned}
$$

B $\mathcal{N}=4$ super Yang-Mills theory on $\mathcal{N}=1/2$ superspace

In this appendix we calculate the Lagrangian of $\mathcal{N}=4$ super Yang-Mills theory defined on $\mathcal{N}=1/2$ superspace. In terms of $\mathcal{N}=1$ superfields, this theory is constructed by a vector superfield $V(x,\theta,\bar\theta)$ and three chiral superfields $\Phi_i(y,\theta)$ $(i=1,2,3)$ which belong to the adjoint representation of the gauge group $U(N)$. The deformed Lagrangian [14] is defined by

$$
\begin{aligned}
\mathcal{L}_c^{\mathcal{N}=4} &= \frac{1}{k}\int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\sum_{i=1}^{3}\left(\overline{\Phi}_i * e^V * \Phi_i * e^{-V}\right) \\
&\quad + \frac{1}{16kg_{\mathrm{YM}}^2}\int d^2\theta\ \mathrm{Tr}\left(W^\alpha * W_\alpha\right) + \frac{1}{16kg_{\mathrm{YM}}^2}\int d\bar{\theta}\ \mathrm{tr}\left(\overline{W}_{\dot{\alpha}} * \overline{W}^{\dot{\alpha}}\right) \\
&\quad - \frac{\sqrt{2}}{3}\frac{g_{\mathrm{YM}}}{k}\int d^2\theta\ \mathrm{Tr}\varepsilon^{ijk}\left(\Phi_i * \Phi_j * \Phi_k\right) + \frac{\sqrt{2}}{3}\frac{g_{\mathrm{YM}}}{k}\int d^2\bar{\theta}\ \mathrm{Tr}\varepsilon^{ijk}\left(\overline{\Phi}_i * \overline{\Phi}_j * \overline{\Phi}_k\right).
\end{aligned}
\tag{76}
$$

Here the star product is defined by $f(\theta) * g(\theta) = f(\theta)\exp\left[-\frac{1}{2}C^{\alpha\beta}\overleftarrow{Q_\alpha}\overrightarrow{Q_\beta}\right] g(\theta)$. Q_α is the supercharge defined on the superspace. It is convenient to redefine the component fields of a superfield such that they transform canonically under the gauge transformation. The expansion of the chiral superfield is the same as the undeformed one:

$$
\Phi_i(y,\theta) = \phi_i(y) + i\sqrt{2}\theta\psi_i(y) + \theta\theta F_i(y). \tag{77}
$$

The anti-chiral superfield is expanded as [22]

$$
\overline{\Phi}_i(\bar{y},\bar{\theta}) = \bar{\phi}_i(\bar{y}) + i\sqrt{2}\bar{\theta}\bar{\psi}_i(\bar{y}) + \bar{\theta}\bar{\theta}\left(\bar{F}_i(\bar{y}) + iC^{\mu\nu}\partial_\mu\{\bar{\phi}_i, A_\nu\}(\bar{y}) - \frac{g_{\mathrm{YM}}}{2}C^{\mu\nu}[A_\mu, \{A_\nu, \bar{\phi}_i\}](\bar{y})\right) \tag{78}
$$

where we have defined $C^{\mu\nu} = C^{\alpha\beta}\varepsilon_{\beta\gamma}(\sigma^{\mu\nu})_\alpha{}^\gamma$. The vector superfield in the Wess-Zumino gauge is [4]

$$
\begin{aligned}
V(y,\theta,\bar{\theta}) &= -\theta\sigma^\mu\bar{\theta}A_\mu(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - i\bar{\theta}\bar{\theta}\theta^\alpha\left(\lambda_\alpha(y) + \frac{1}{4}\varepsilon_{\alpha\beta}C^{\beta\gamma}\sigma^\mu_{\gamma\dot{\gamma}}\{\bar{\lambda}^{\dot{\gamma}}, A_\mu\}(y)\right) \\
&\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D(y) - i\partial_\mu A^\mu(y)\right).
\end{aligned}
\tag{79}
$$

Rescaling appropriately component fields and $C^{\alpha\beta}$ by gauge coupling constant $g_{\rm YM}$, we find that Lagrangian (76) becomes

$$\begin{aligned}
\mathcal{L}_c^{\mathcal{N}=4} &= \frac{1}{kg_{\rm YM}^2}\mathrm{Tr}\Bigg[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}-D^\mu\bar{\phi}_iD_\mu\phi_i+\bar{F}_iF_i+\frac{1}{2}D^2\\
&\quad -i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i-i\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda-i\sqrt{2}[\bar{\phi}_i,\psi_i]\lambda-i\sqrt{2}[\phi_i,\bar{\psi}_i]\bar{\lambda}+D[\phi_i,\bar{\phi}_i]\\
&\quad -\frac{i}{2}C^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda}+\frac{1}{8}|C|^2(\bar{\lambda}\bar{\lambda})^2+\frac{i}{2}C^{\mu\nu}F_{\mu\nu}\{\bar{\phi}_i,F_i\}\\
&\quad -\frac{\sqrt{2}}{2}C^{\alpha\beta}\{D_\mu\bar{\phi}_i,(\sigma^\mu\bar{\lambda})_\alpha\}\psi_{i\beta}-\frac{1}{16}|C|^2[\bar{\phi}_i,\lambda][\bar{\lambda},F_i]\\
&\quad -\sqrt{2}\varepsilon^{ijk}\left(F_i\phi_j\phi_k-\phi_i\psi_j\psi_k-\frac{1}{12}|C|^2F_iF_jF_k-\frac{1}{2}C^{\alpha\beta}F_i\psi_{j\alpha}\psi_{k\beta}\right)\\
&\quad +\sqrt{2}\varepsilon^{ijk}\left(\bar{F}_i\bar{\phi}_j\bar{\phi}_k-\bar{\phi}_i\bar{\psi}_j\bar{\psi}_k+\frac{2i}{3}C^{\mu\nu}F_{\mu\nu}\bar{\phi}_i\bar{\phi}_j\bar{\phi}_k+\frac{1}{3}C^{\mu\nu}D_\mu\bar{\phi}_iD_\nu\bar{\phi}_j\bar{\phi}_k\right)\Bigg].
\end{aligned} \tag{80}$$

Integrating out the auxiliary fields we get

$$\begin{aligned}
\mathcal{L}_c^{\mathcal{N}=4} &= \frac{1}{kg_{\rm YM}^2}\mathrm{Tr}\Bigg[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}-i\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda-i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i-D^\mu\bar{\phi}_iD_\mu\phi_i\\
&\quad -i\sqrt{2}[\bar{\phi}_i,\psi_i]\lambda-i\sqrt{2}[\phi_i,\bar{\psi}_i]\bar{\lambda}-\frac{1}{2}[\phi_i,\bar{\phi}_i]^2+[\bar{\phi}_i,\bar{\phi}_j][\phi_i,\phi_j]\\
&\quad -\frac{i}{2}C^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda}+\frac{1}{8}|C|^2(\bar{\lambda}\bar{\lambda})^2-\frac{\sqrt{2}}{2}C^{\alpha\beta}\{D_\mu\bar{\phi}_i,(\sigma^\mu\bar{\lambda})_\alpha\}\psi_{i\beta}\\
&\quad -\sqrt{2}\varepsilon^{ijk}\left(-\phi_i\psi_j\psi_k+\bar{\phi}_i\bar{\psi}_j\bar{\psi}_k+\frac{i}{3}C^{\mu\nu}F_{\mu\nu}\bar{\phi}_i\bar{\phi}_j\bar{\phi}_k-\frac{1}{3}C^{\mu\nu}D_\mu\bar{\phi}_iD_\nu\bar{\phi}_j\bar{\phi}_k\right)\\
&\quad -C^{\alpha\beta}[\bar{\phi}_i,\bar{\phi}_j]\psi_{i\alpha}\psi_{j\beta}+\frac{\sqrt{2}}{16}|C|^2\varepsilon^{ijk}[\bar{\phi}_i,\lambda][\bar{\lambda},\bar{\phi}_j\bar{\phi}_k]\\
&\quad +\frac{1}{12}|C|^2\varepsilon^{ipq}\varepsilon^{jrs}[\bar{\phi}_i,\bar{\phi}_j][\bar{\phi}_p,\bar{\phi}_q][\bar{\phi}_r,\bar{\phi}_s]\Bigg].
\end{aligned} \tag{81}$$

We note that the term $-C^{\alpha\beta}[\bar{\phi}_i,\bar{\phi}_j]\psi_{i\alpha}\psi_{j\beta}$ in the 5th line is absent in [14]. The relation between scalar fields φ_a and ϕ_i is given by

$$\varphi_{2i-1}=\frac{1}{\sqrt{2}}\left(\phi_i+\bar{\phi}_i\right),\quad \varphi_{2i}=\frac{i}{\sqrt{2}}\left(\phi_i-\bar{\phi}_i\right),\quad (i=1,2,3). \tag{82}$$

References

[1] H. Ooguri and C. Vafa, Adv. Theor. Math. Phys. **7** (2003) 53, hep-th/0302109; Adv. Theor. Math. Phys. **7** (2004) 405, hep-th/0303063.

[2] J. de Boer, P. A. Grassi and P. van Nieuwenhuizen, Phys. Lett. **B574** (2003) 98, hep-th/0302078.

[3] N. Berkovits and N. Seiberg, JHEP **0307** (2003) 010, hep-th/0306226.

[4] N. Seiberg, JHEP **0306**, 010 (2003), hep-th/0305248.

[5] M. Billó, M. Frau, I. Pesando and A. Lerda, JHEP **0405** (2004) 023, hep-th/0402160.

[6] E. Ivanov, O. Lechtenfeld and B. Zupnik, JHEP **0402** (2004) 012, hep-th/0308012; Nucl. Phys. B **707** (2005) 69, hep-th/0408146.

[7] T. Araki, K. Ito and A. Ohtsuka, JHEP **0401** (2004) 046, hep-th/0401012; Phys. Lett. **B606** (2005) 202, hep-th/0410203.

[8] S. Ferrara, E. Ivanov, O. Lechtenfeld, E. Sokatchev and B. Zupnik, Nucl. Phys. B **704** (2005) 154, hep-th/0405049.

[9] T. Araki and K. Ito, Phys. Lett. **B595** (2004) 513, hep-th/0404250.

[10] A. De Castro, E. Ivanov, O. Lechtenfeld, L. Quevedo, Nucl. Phys. **B747** (2006) 1, hep-th/0510013;
A. De Castro and L. Quevedo Phys. Lett. **B639** (2006) 117, hep-th/0605187.

[11] K. Ito and S. Sasaki, JHEP **0611** (2006) 004, hep-th/0608143.

[12] R. C. Myers, JHEP **9912** (1999) 022, hep-th/9910053.

[13] M. R. Garousi and R. C. Myers, JHEP **0011** (2000) 032, hep-th/0010122.

[14] A. Imaanpur, JHEP **0503** (2005) 030, hep-th/0501167;
R. Abbaspur and A. Imaanpur, JHEP **0601** (2006) 017, hep-th/0509220.

[15] M. Billo, M. Frau, I. Pesando, F. Fucito, A. Lerda, A. Liccardo, JHEP **0302** (2003) 045, hep-th/0211250.

[16] V. A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, Nucl. Phys. **B288** (1987) 173.

[17] D. Friedan, E. J. Martinec and S. H. Shenker, Nucl. Phys. **B271** (1986) 93.

[18] P. Di Vecchia, L. Magnea, A. Lerda, R. Russo and R. Marotta, Nucl. Phys. **B469** (1996) 235, hep-th/9601143.

[19] M. Billó, M. Frau, F. Fucito and A. Lerda, JHEP **0611** (2006) 012, hep-th/0606013.

[20] N. Berkovits, JHEP **0004** (2000) 022, hep-th/0001084.

[21] A. S. Losev, A. Marshakov and N. A. Nekrasov, hep-th/0302191;
N. A. Nekrasov and A. Okounkov, hep-th/0306238.

[22] T. Araki, K. Ito and A. Ohtsuka, Phys. Lett. **B573** (2003) 209, hep-th/0307076;
K. Ito and H. Nakajima, Phys. Lett. **B633** (2006) 776, hep-th/0511241